Exponentiation

Exponentiation is a mathematical operation, written as x^n, involving two numbers, the base x and the exponent or power n. When n is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, x^n is the product of multiplying n bases: $x^n = x * x * \ldots * x$.

How to find this value if x and n are given? We can use just simple loop with complexity O(n) like

```
res = 1;
for (i = 1; i <= n; i++)
  res = res * x;
```

Can we do it faster? For example, $x^{10} = (x^5)^2 = (x * x^4)^2 = (x * (x^2)^2)^2$.
We can notice that $x^{2n} = (x^2)^n$, or $x^{100} = (x^2)^{50}$.

$$x^n = \begin{cases} \left(x^2\right)^{n/2}, n \text{ is even} \\ x \cdot x^{n-1}, n \text{ is odd} \\ 1, n = 0 \end{cases}$$

```
int f(int x, int n)
{
  if (n == 0) return 1;
  if (n % 2 == 0) return f(x * x, n / 2);
  return x * f(x, n - 1);
}
```

Complexity O($\log_2 n$).

E-OLYMP 5198. Modular Exponentiaion Find the value of x^n mod m.
► Use the function f(x, n, m) = x^n mod m.

```
long long f(long long x, long long n, long long m)
{
  if (n == 0) return 1;
  if (n % 2 == 0) return f((x * x) % m, n / 2, m);
  return (x * f(x, n - 1, m)) % m;
}
```

E-OLYMP 9644. Sum of powers Find the value of the sum

$$(1^n + 2^n + 2 * 3^n + 3 * 4^n + 4 * 5^n + ... + 99 * 100^n) \bmod m$$

► The first two terms of the sum differ from the rest. Let's calculate them separately. Next, we calculate the sum in a loop of i from 3 to 100, each term has the form $(i - 1) * i^n$.

E-OLYMP [9557. Bins and balls](#) There are n bins in a row. There is also an infinite supply of balls of n distinct colors. Place exactly one ball into each bin, with the restriction that adjacent bins cannot contain balls of the same color. How many valid configurations of balls in bins are there?

► Any of n balls can be put into the first box. The color of the ball in the second box must not match the color of the ball in the first box. Therefore, you can put any ball of $n - 1$ colors in the second box. In the i-th box, you can put a ball of any color that does not match the color of the ball in the $(i - 1)$-th box.

1	2	3	4		n
n	n-1	n-1	n-1	...	n-1

Thus, the number of different arrangements of balls in the boxes equals to

$$n * (n - 1)^{n-1} \bmod 10^9 + 7$$